

The cost-efficiency carbon pricing puzzle*

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Abstract

Any global temperature target must be translated into an intertemporal carbon budget and its associated cost-efficient carbon price schedule. Under the Hotelling's rule, the growth rate of this price should be equal to the interest rate. It is therefore a puzzle that cost-efficiency IAM models yield carbon prices that increase at an average real growth rate around 7% per year. This carbon pricing puzzle suggests that their abatement trajectories are not intertemporally optimized, probably because of the political unacceptability of a high initial carbon price. Using an intertemporal asset pricing approach, I examine the impact of the uncertainties surrounding economic growth and abatement technologies on the dynamics of efficient carbon prices, interest rates and risk premia. I show that marginal abatement costs and aggregate consumption are positively correlated along the optimal abatement path, implying a positive carbon risk premium and an efficient growth rate of expected carbon prices larger than the interest rate. From this numerical exercise, I recommend a growth rate of expected carbon price around 3.75% per year (plus inflation). I also show that the rigid carbon budget approach to cost-efficiency carbon pricing implies a large uncertainty surrounding the future carbon prices that support this constraint. In this model, green investors are compensated for this risk by a large risk premium embedded in the growth rate of expected carbon prices, not by a collar on carbon prices as often recommended.

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JEL codes: Q54, D81, G12.

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1 Introduction

How urgent is the necessity to decarbonize our economies? Can we wait another decade to get the anticipated low-cost low-carbon technologies before asking people to finance the energy transition? Should we again postpone the drastic increase in carbon price necessary to trigger this transition? In this paper, I address these key policy questions by recognizing that politicians have already fixed the climate objective of 2°C without knowing the cost of the green technologies that one will have to use in the next few decades to attain this ambitious objective. This ambition has been confirmed at the occasion of the COP-21 in Paris in 2015. As is well-known, it is associated to an intertemporal carbon budget constraint. Determining the optimal timing to consume this carbon budget is a problem isomorphic to the Hotelling's problem (Hotelling (1931)) of extracting a non-renewable resource (Chakravorty et al. (2006), Chakravorty et al. (2008), Schubert (2008)). Under this cost-efficiency approach, abating one ton of CO_2 today is a perfect substitute to abating one ton of CO_2 in the future.¹ Frontloading the abatement effort is an investment that has a single cost and a single benefit that are respectively equal to the present and future marginal abatement costs (MAC), i.e., to the present and future carbon prices. Along the optimal abatement path, this marginal investment should have a zero net present value. This is possible only if the growth rate of (expected) carbon price is equal to the (risk-adjusted) discount rate. This extended Hotelling's rule applied to climate change is simple and transparent.² The ambition of the climate target or the anticipation of future low-cost abatement technologies should influence the initial carbon price, but not its growth rate over time. In short, under an exogenous climate objective, the Hotelling's rule dictates the efficient timing of our climate efforts.

In most climate models, there is no uncertainty and green technological progresses are known in advance. In that case, the growth rate of carbon prices should therefore be equal to the interest rate.³ It is then a puzzle that most of these models generate carbon prices whose real growth rate is much larger than the interest rate. Figure 1 illustrates this observation. It describes the distribution of annualized real growth rates of world carbon prices from the database (<https://tntcat.iiasa.ac.at/AR5DB>) of models used in the 5th report of the IPCC. If one limits the analysis to the 767 calibrations of these models that estimate a world carbon prices for years 2020 and 2050, they yield an average annual growth rate of 7.04% for real carbon prices between these two dates, which is much larger than market interest rates. This suggests that the allocation of mitigation efforts is not intertemporally efficient. I refer to this observation as the "carbon pricing puzzle" of cost-efficient IAM models. It tells us that, compared to the recommendations extracted from these IPCC

¹In this introduction, I ignore the natural decay of GHG in the atmosphere.

²It is specific to the cost-efficient approach and does not need to hold in the cost-benefit approach used for example by Nordhaus (2018). For example, using a 3% discount rate, the U.S. administration published a scientific report (IAWG (2016)) based on a cost-benefit approach that recommends a price of 42 dollars (of 2007) per ton of CO_2 in 2020, growing to 69 dollars (of 2007) in 2050. This yields a real growth rate of 1.65% per year. Because the carbon concentration in the atmosphere will continue to grow over time under the optimal mitigation strategy, carbon prices will grow in parallel, assuming a convex damage function.

³In the absence of any credibility problem, the decentralization of the allocation of the intertemporal carbon budget should be performed by allocating the corresponding permits in the economy, allowing for banking. Under certainty, these permits are risk-free, which implies that their value – the carbon price – should grow at the risk-free rate. Attempting to impose a larger growth rate will generate a disequilibrium (excess saving of permits).

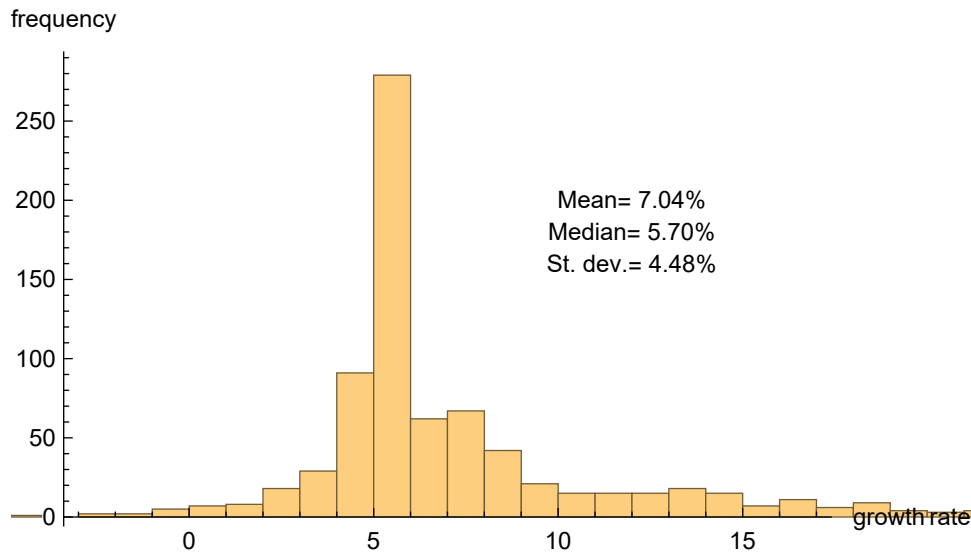


Figure 1: Histogram of the real growth rate (in % per year) of carbon prices between 2020 and 2050 from 767 calibrations of IAM models contained in the IPCC database (<https://tntcat.iiasa.ac.at/AR5DB>). The mean annual growth rate of 7.04%, with a median of 5.70% and a standard deviation of 4.48%.

models, reallocating some climate efforts to the present would be socially desirable. In reality, these models explore second-best climate policies in which the intertemporal allocation of the carbon budget is not optimized, certainly because of the political unacceptability of a high carbon price. Rather, these models characterize carbon price schedules that are compatible with exogenously determined carbon emission targets at different dates. These "Representative Concentration Pathways" (RCP) are predetermined by the IPCC. The large growth rate of carbon prices suggests that the waiting game of international climate politics has infected the IPCC.⁴

In the United States, the Climate Leadership Council (CLC) has sponsored an "Economists' Statement on Carbon Dividends" in early 2019. The CLC supports a carbon price at 40 USD₂₀₁₇ growing at 5% per year above inflation. In France, a recent public report (Quinet (2019)) has recommended a carbon price growing from 69 EUR₂₀₁₉ to 775 EUR₂₀₁₉ between 2020 and 2050, yielding a real growth rate of 8% per annum. In the United Kingdom, the official public carbon values grow from 14 GBP₂₀₁₈ in 2020 to 81 GBP₂₀₁₈ in 2030 (BEIS (2019)), implying a growth rate of 16% per annum. The carbon pricing puzzle described in this paper exceeds by far the scientific circle around the IPCC and the community of IAM modelers.

However, this initial puzzle is based on the premise that the evolution of abatement costs and carbon prices is certain. In this paper, I recognize that this key assumption is utterly unrealistic, and I explore the impact of uncertainty on the socially efficient growth rate of real

⁴It is a fair question to ask whether the IPCC should base its recommendations on the first-best allocation, or whether it should include the political acceptability constraint straight from the beginning into its analysis. My view on this is that second-best analyses are useful as long as they are clearly announced as such.

carbon prices. In particular, I explore the possibility that the puzzle be solved by introducing risk into the model. From a positive viewpoint, I predict the growth rate of expected carbon prices if the intertemporal carbon budget is decentralized through a market for permits with full banking.

The abatement models using a cost-efficiency approach and a carbon budget rely on strong assumptions about the evolution of the abatement cost function during the next few decades (Pindyck (2013)). Obviously, technologically optimistic models allow for low prices and efforts in the short run by anticipation of the emergence of these low-cost mitigation technologies. But in reality, technological changes are hard to predict. If they do not materialize, one will have to drastically increase carbon prices to satisfy the intertemporal carbon budget. Nobody really knows today what will be the mitigation cost associated to wind or solar energy in the future. Deep uncertainties also surround future electricity storage technologies and nuclear fusion for example. The extraordinary large uncertainty surrounding the emergence of economically viable renewable systems of energy is an inherent dimension of the energy transition. Similarly, IAM models are generally based on a deterministic growth of total factor productivity. Recognizing the uncertainty surrounding the growth of TFP in the long run should also be taken into account to determine the carbon price schedule. If economic growth is larger than expected, more abatement efforts will have to be implemented to compensate for the larger emissions and this will require a larger carbon price. As in the "quantity" approach proposed by Weitzman (1974) under uncertainty, I assume that the carbon budget is not sensitive to changes in the marginal abatement costs.

Uncertainty should affect the optimal timing of climate efforts and the carbon pricing system that support it.⁵ The expected growth rate of carbon prices – which is also the expected return of abatement frontloading – should equal the discount rate adjusted for the riskiness of postponing or frontloading the abatement effort. The Consumption-based Capital Asset Pricing Model (CCAPM, Breeden (1979), Lucas (1978) and Rubinstein (1976)) tells us how to perform this adjustment. Suppose for example that, along the optimal path, marginal abatement costs are negatively correlated with aggregate consumption. Because the MAC is the future benefit of abatement frontloading, fighting climate change early has the extra benefit to hedge the macro risk in that context. Because of this negative CCAPM beta of early mitigation efforts, one should discount the future benefit of this early investment, i.e., the future MAC, at a rate lower than the risk-free rate to determine the current price of carbon. This means at the same time a larger current price of carbon, and a growth rate of the expected carbon price smaller than the risk-free rate. From a positive point of view, this carbon pricing system is compatible with an equilibrium, as investors in green technologies will have an expected rate of return smaller than the interest rate, just because such green investments hedge their global portfolio risk. On the contrary, if MAC and aggregate con-

⁵The theoretical question raised here is about how to adapt the Hotelling's rule to uncertainty. There has been a few attempts to answer this question in the late XXth century. For example, Pindyck (1978, 1980) explores the optimal extraction strategy of risk-neutral owners of a nonrenewable resource when exploration is possible or when the stock of this resource and the demand for it are unknown. This analysis is useful to examine a resource-rich country that is unable or unwilling to make this asset financially liquid, but it is not directly relevant in the context of the carbon budget problem. Indeed, households, investors and firms that will bear the mitigation risk will also bear all other statistically-linked risks in the economy. Our approach is closer to Gaudet and Howitt (1989), Gaudet and Khadr (1991) and Slade and Thille (1997) who examined the case of stochastic processes for economic growth and extraction costs in the context of a non-renewable resource.

sumption correlate positively, i.e., if the climate beta is positive, the risk premium will be positive, the current price of carbon will be smaller, and the growth rate of expected carbon price will be larger than the interest rate. This policy provides the right price signal for private investors in renewables technologies to take account of the impact of their decisions on social welfare, as is the case on efficient financial markets for other investment projects.

It remains to characterize the determinants of this carbon beta.⁶ To do this, I develop a two-period "act-then-learn" model in which the dynamically optimal mitigation strategy is endogenously determined under uncertainty about the future abatement cost function, economic growth and carbon budget. I characterize the impact of these sources of uncertainty on the optimal growth rate of expected carbon price, and I realistically calibrate this model. Most integrated assessment models which allow for uncertainty do that by using a "learn-then-act" methodology. Under that approach, it is assumed that the modeler observes the realization of the vector of uncertain parameters before optimizing the climate policy under certainty. The mean value of the conditionally optimal policies across all possible realizations of this vector is then recommended as the optimal policy under uncertainty. This method is not satisfactory because it ignores the timing of the resolution of the uncertainty, and therefore the role of precaution that is inherent to our world. It also produces carbon prices that are not coherent with the system of assets prices in the economy. By using stochastic dynamic optimization and backward induction, I more realistically determine the optimal climate policy in the first period before the resolution of the uncertainty taking place in the second period. I solve the classical asset pricing puzzles (Mehra and Prescott (1985), Weil (1989) and Kocherlakota (1996)) of the CCAPM by introducing catastrophes in the growth process, as suggested by Barro (2006).⁷ In this framework, I show that the beta of abatement frontloading is the income-elasticity of MACs. Multiplying this beta by the equilibrium aggregate risk premium tells us by how much the growth rate of expected carbon price should differ from the equilibrium interest rate. I show that the sign of this carbon beta is generally ambiguous, with different sources of uncertainty pushing the climate beta in opposite directions. However, a realistic calibration of the two-period model suggests a positive climate beta. This means that it is socially desirable to implement a climate strategy with a growth rate of expected carbon price that is larger than the interest rate, thereby allowing to start with a relatively low carbon price today. Thus, this analysis justifies using a discount rate for green technologies and planning for a growth rate of expected carbon prices that are larger than the interest rate. It could thus help solving the carbon pricing puzzle. However, the efficient growth rate of carbon prices is around 3.5%, which is much smaller than the 7.04% observed on average in the database of models of the IPCC. The bottom line of my analysis remains that the RCPs of the IPCC inefficiently allocate abatement efforts over time. The same final concentration of GHG in the atmosphere could be obtained with a smaller impact on intergenerational welfare by abating more today, and abating less in the future.

A possible explanation of the carbon pricing puzzle is based on the existence of political constraints related to the social acceptability of climate policies around the world in the short

⁶Dietz et al. (2018) examined the risk profile of carbon prices using the cost-benefit analysis of the DICE model. In this alternative approach, the key determinant of the climate beta is the income-elasticity of climate damages.

⁷I also examine a model in which the asset pricing puzzles are resolved by using the long run risk approach of Bansal and Yaron (2004). I show that my key results are not affected by this critical change in modeling.

run. Following Gollier and Tirole (2015) for example, these constraints are typically at play to postpone climate efforts to the future, a phenomenon of procrastination that could explain why the above-mentioned models support a low current carbon price and a large growth rate of this price. This raises the question of the credibility of long-term climate commitments. Laffont and Tirole (1996) take this question seriously by proposing a commitment device based on forward financial contracts. Harstad (2019) justifies strategic investments and investment subsidies in technologies that are strategic complements to future green investments when the social planner faces a time-consistency problem from hyperbolic discounting.

In the next section, I assume that the optimal abatement strategy under the carbon budget is known, and I characterize the properties of the carbon pricing system that supports this social optimum, assuming an exogenous statistical relation between MAC and aggregate consumption. Section 3 is devoted to a simple two-period model in which the price of carbon in the first period must be determined under uncertainty about economic growth, green innovation and carbon budget. The carbon beta is determined endogenously in this section. In Section 4, I calibrate this model.

2 CCAPM carbon pricing

In this section, I characterize the socially optimal expected growth rate of the carbon price based on the classical consumption-based CAPM model. In the spirit of this model, the optimality condition is translated into an asset pricing rule. This rule can be used as an optimality test for the underlying dynamic allocation. However, it provides only a partial characterization of the optimal allocation. Its full characterization is provided in the next section in a simplified framework.

Suppose that the economy has a representative agent whose rate of pure preference for the present is ρ . The von Neumann-Morgenstern utility function u of the representative agent is increasing and concave. Along the optimal path, the consumption per capita $C_\tau|_{\tau \geq 0}$ evolves in a stochastic way. In the constellation of investment opportunities existing in the economy, consider a marginal incremental project that yields a cost I_0 today and generates a single benefit B_t at date t , where B_t is potentially uncertain and statistically related to the stochastic process governing aggregate consumption. At the margin, investing in this project raises the discounted expected utility of the representative agent by

$$\Delta V = -I_0 u'(C_0) + e^{-\rho t} E[B_t u'(C_t)]. \quad (1)$$

The size of the investment in this project is optimal if and only if $\Delta V = 0$. If one reinterprets I_0 as the current price of an asset yielding the single benefit B_t at date t , this optimality condition is also an equilibrium condition and an asset pricing rule. The CCAPM makes use of this observation to price any asset in the economy. For example, the risk-free claim $B_t = 1$ should be priced today as $P_{ft} = e^{-\rho t} E[u'(C_t)]/u'(C_0)$. This equilibrium condition gives us the interest rate r_{ft} in the economy once price P_{ft} is translated into a return $r_{ft} = -t^{-1} \log(P_{ft})$.

Let $A'_\tau|_{\tau \geq 0}$ denote the dynamics of marginal abatement costs along the optimal allocation of mitigation efforts. It can also be interpreted as the dynamics of equilibrium carbon prices. If the climate policy is decentralized through a market for bankable emission permits for example, marginal abatement costs will be equalized across firms and individuals, and will be equal to the equilibrium carbon price. Let δ denote the rate of natural decay of CO₂

in the atmosphere. Along the optimal mitigation path, contemplate a marginal reallocation of climate efforts consisting in reducing CO₂ emissions by 1 more ton today. This allows for a marginal increase in emissions by $\exp(-\delta t)$ tons at date t , leaving the total carbon budget unaffected. This investment yields an initial cost A'_0 and generates a future benefit $B_t = \exp(-\delta t)A'_t$. Applying the optimality condition $\Delta V = 0$ to this strategy, we must have that

$$A'_0 = \exp(-(\delta + \rho)t) \frac{E[A'_t u'(C_t)]}{u'(C_0)}. \quad (2)$$

Suppose that A'_t and C_t are comonotone.⁸ Because of risk aversion, equation (2) implies that (see for example Gollier (2001), Proposition 15):

$$\frac{A'_0}{E[A'_t]} < \exp(-(\delta + \rho)t) \frac{E[u'(C_t)]}{u'(C_0)} = \exp(-(r_{ft} + \delta)t). \quad (3)$$

Let $g_t = t^{-1} \log(E[A'_t]/A'_0)$ denote the growth rate of expected carbon price.⁹ The above inequality directly implies that this growth rate is larger than $r_{ft} + \delta$. The opposite is true when A'_t and C_t are anti-comonotone. In the special case where A'_t is certain, the growth rate of carbon price should be equal to the sum of the interest rate and the rate of natural decay. This is the well-known Hotelling's rule adapted to carbon pricing under a fixed intertemporal carbon budget (Schubert (2008)). This proves the following proposition.

Proposition 1. *The growth rate of the expected carbon price that supports the optimal temporal allocation of abatement efforts is larger (smaller) than the sum of the interest rate and the rate of decay of carbon dioxide if the marginal abatement cost and aggregate consumption are (anti-)comonotone.*

From the social point of view, facing a positive correlation between marginal abatement costs and aggregate consumption is good news. It means that the worst-case scenarios in terms of abatement costs arise when aggregate consumption is large, i.e., when the marginal abatement effort has a smaller utility impact. Abating more in the future reduces the macroeconomic risk. This hedging benefit raises the collective willingness to postpone abatement efforts. It reduces the efficient carbon price today, in exchange for a larger growth rate of the expected price. From the individual point of view, investors who abate early in exchange for saving their permits must be compensated for the fact that the benefit of doing so has a positive beta, in the sense that the return of this investment is smaller when other assets also perform poorly in the economy. Because the return of abatement frontloading is the growth rate of carbon price, this compensation takes the form of a growth rate of expected carbon price larger than the sum of the interest rate and the rate of natural decay.

I illustrate Proposition 1 in two special cases. The benchmark case corresponds to the standard CCAPM. Suppose that relative risk aversion is a constant γ . Suppose also that aggregate consumption and marginal abatement costs evolve according to the following stochas-

⁸Two random variables (X, Y) are said to be comonotone iff for any pair (s, s') of states of nature, $(X(s) - X(s'))(Y(s) - Y(s'))$ is non-negative. Anti-comonotonicity is defined symmetrically.

⁹In general, variables g_t and r_{ft} are maturity-dependent. Our findings should be understood as being applicable to any maturity.

tic process:

$$dc_t = \mu_c dt + \sigma_c dz_t \quad (4)$$

$$da'_t = \mu_p dt + \phi \sigma_c dz_t + \sigma_w dw_t, \quad (5)$$

with $c_t = \log C_t$ and $a'_t = \log A'_t$, and where z_t and w_t are two independent standard Wiener processes. This means that the logarithm of aggregate consumption and marginal costs are jointly normally distributed. Parameters μ_c and σ_c are respectively the trend and the volatility of consumption growth. The trend of growth of the marginal abatement cost, and thus of the carbon price, is given by parameter μ_p . The volatility of the marginal abatement cost has an independent component σ_w and a component coming from its correlation with economic growth. Notice that ϕ can be interpreted as the elasticity of marginal abatement costs to unanticipated changes in aggregate consumption.

I provide a formal proof of the following proposition in the Appendix, together with the characterization of the risk-free rate and the aggregate risk premium. It is an application of the CCAPM, and to its illustration to the pricing a non-renewable resource (Gaudet and Khadr (1991)).

Proposition 2. *Suppose that relative risk aversion is constant and that the logarithms of aggregate consumption and marginal abatement costs follow a bivariate Brownian process. Then, the growth rate of the expected carbon price that supports the optimal temporal allocation of abatement efforts must be equal to the sum of three terms:*

- δ : the rate of natural decay of greenhouse gas in the atmosphere;
- r_f : the interest rate in the economy;
- $\phi\pi$: the abatement risk premium, which is the product of the income-elasticity (ϕ) of marginal abatement cost by the aggregate risk premium (π) in the economy.

In short, we have that

$$g = \delta + r_f + \phi\pi, \quad (6)$$

where the interest rate r_f and the aggregate risk premium π are characterized in the Appendix. This result tells us that the CCAPM risk premium for carbon permits holds with a CCAPM "carbon beta" being equal to the income-elasticity ϕ of the marginal abatement cost. An immediate consequence of Proposition 2 is that the growth rate of expected carbon price is larger (smaller) than the sum of the interest rate and the rate of decay of carbon dioxide if the income-elasticity of marginal abatement costs is positive (negative). This is a special case of Proposition 1.

Under the stochastic process (4)-(5), the estimation of the key parameter ϕ is rather simple. Indeed, this system implies that

$$\Delta \log(A'_t) = a + \phi \Delta \log(C_t) + \varepsilon_t, \quad (7)$$

where $\Delta \log(A'_t)$ and $\Delta \log(C_t)$ are respectively changes in log marginal cost and in log consumption, and ε_t is an independent noise that is normally distributed. This means that, under these assumptions, the OLS estimator of the slope of this linear equation is an unbiased estimator of the income-elasticity of the marginal abatement cost that must be used to determine the efficient growth rate of expected carbon price.

This CCAPM example is imperfect for at least two reasons. First, as is well-known, the CCAPM faces the standard puzzles of asset pricing, in particular the risk-free rate puzzle and the equity premium puzzle. Second, it is clear that the income-elasticity of the marginal abatement cost is endogenous and sensitive to the mitigation strategy that will be followed at equilibrium. This is why the remainder of this paper is devoted to the analysis of an alternative application of Proposition 1 that solves these two issues.

3 The determinants of the carbon beta under an exogenous carbon budget

In this section, I explore the determinants of the income-elasticity of the marginal abatement cost, i.e., the cost-efficient carbon beta. Because the current and future marginal abatement costs depend upon the intertemporal abatement strategy, its characterization requires solving the intertemporal carbon allocation problem. This cannot be easily done in a continuous-time framework. In this section, I solve this problem in a simple two-period framework. Suppose that the carbon budget constraint covers only two periods, $t = 0$ and 1. The production of the consumption good is denoted Y_0 and Y_1 for periods 0 and 1 respectively, where Y_1 is uncertain in period 0. The carbon intensity of the economy in the business-as-usual scenario in period t is denoted Q_t , so that $Q_t Y_t$ tons of carbon dioxide are emitted in period t under this scenario. The country is committed not to exceed a total emission target T for the two periods, net of the natural carbon sinks. As stated for example in the Paris Agreement, the long-term carbon budget allocated to the countries could be modified depending upon new scientific information about the intensity of the climate change problem for example. In our model, this means that, in period 0, there may be some uncertainty about what the intertemporal carbon budget T will be in the future.

Compared to the business-as-usual scenario, the country must choose how much to abate in each period. Let K_t denote the number of tons of carbon dioxide abated due to actions implemented in period t ,¹⁰ so that one can write the carbon budget constraint as follows:

$$e^{-\delta} (Q_0 Y_0 - K_0) + Q_1 Y_1 - K_1 \leq T, \quad (8)$$

where δ is the rate of natural decay of carbon dioxide in the atmosphere. I hereafter assume that this ex-post carbon budget constraint is always binding, so that I can rewrite the abatement in period 1 as a function of the other variables:

$$K_1 = K_1(K_0, Y_1, T) = e^{-\delta} (Q_0 Y_0 - K_0) + Q_1 Y_1 - T. \quad (9)$$

Because Y_1 and T are uncertain, so is the abatement effort K_1 in period 1 that will be necessary to satisfy the intertemporal carbon budget constraint.

Abating is costly. Let $A_0(K_0)$ and $A_1(K_1, \theta)$ denote the abatement cost function in periods 0 and 1 respectively. I assume that A_t is an increasing and convex function of K_t . In order to allow for technological uncertainty, A_1 is a function of parameter θ , which is unknown in

¹⁰This definition allows us to include in the analysis long-term green investments made in period 0 that also reduce emissions in period 1 at zero marginal cost. Under this interpretation, K_1 is the abatement in period 1 net of the abatement generated by investments made in the previous period.

period 0. Consumption in period t is the production net of the abatement cost in that period, i.e., $C_t = Y_t - A_t$.

The problem of the social planner is thus to select the abatement strategy (K_0, K_1) that maximizes the intertemporal welfare function subject to the carbon budget constraint:

$$\max_{K_0, K_1} H(K_0, K_1) = u(Y_0 - A_0) + e^{-\rho} E[u(Y_1 - A_1)] \quad s.t. \quad (9). \quad (10)$$

The first-order condition of this problem is written as follows:

$$A'_0 u'(C_0) = e^{-\rho-\delta} E[A'_1 u'(C_1)], \quad (11)$$

where A'_t denote the partial derivative of the total abatement cost function with respect to abatement K_t .

We know from Proposition 1 that the growth rate of the expected carbon price is larger (smaller) than the interest rate plus the rate of natural decay when the marginal abatement cost and aggregate consumption are (anti-)comonotone. In the remainder of this section, I examine various special cases that highlight the factors that determine the nature of the statistical relation between these two random variables along the optimal path. To do this, let us fully differentiate A'_1 and C_1 with respect to the three sources of uncertainty (Y_1, θ, T) :

$$dA'_1 = Q_1 A''_1 dY_1 - A''_1 dT + \frac{\partial A'_1}{\partial \theta} d\theta \quad (12)$$

$$dC_1 = (1 - Q_1 A'_1) dY_1 + A'_1 dT - \frac{\partial A_1}{\partial \theta} d\theta. \quad (13)$$

Suppose first that the only source of uncertainty in the economy is related to the exogenous growth of production Y_1 , so that T and θ are fixed. In that context, the only source of correlation between A'_1 and C_1 comes from the fact that both random variables covary with Y_1 . From the first term in the right-hand side of equation (12), we see that A'_1 is increasing in Y_1 , since A''_1 is positive. A positive productivity shock raises the marginal abatement cost. This is because it raises emissions under the business-as-usual together with the abatement effort to compensate it. Because the MAC is increasing in the effort, it covaries positively with Y_1 . Suppose now that $Q_1 A'_1$ is smaller than unity. From the first term in the right-hand side of equation (13), this implies that a positive productivity shock increases consumption in spite of the fact that it also necessitates an additional abatement effort to compensate the excess emissions generated by the shock. Thus, under this condition, a positive productivity shock affects positively the MAC and aggregate consumption. Thus, A'_1 and C_1 are comonotone in this context. Using Proposition 1, this demonstrates the following proposition.

Proposition 3. *Suppose that the growth of aggregate production Y_1 is the only source of uncertainty in the economy, and that $Q_1 A'_1$ is smaller than unity. Then, it is socially desirable that the growth rate of expected carbon price be larger than the sum of the interest rate and the rate of decay of CO_2 .*

A similar exercise based on the signs of the second terms in equations (12) and (13) can be done in a context where the only source of uncertainty is related to the intertemporal budget constraint T . In that case, a larger budget T implies a smaller abatement effort, and thus a larger share of production available for consumption rather than for abatement efforts.

At the same time, because of the convexity of the cost function, the marginal abatement cost is smaller. Thus, aggregate consumption and marginal abatement cost are anti-comonone. This yields the following result.

Proposition 4. *Suppose that the intertemporal carbon budget T is the only source of uncertainty in the economy. Then, it is socially desirable that the growth rate of expected carbon price be smaller than the sum of the interest rate and the rate of decay of CO_2 .*

Suppose finally that the only source of uncertainty is about θ , which is related to the speed of green technological progress. Suppose that an increase in θ implies a reduction in both the total and the marginal abatement costs, i.e., that for all (K_1, θ) ,

$$\frac{\partial A_1(K_1, \theta)}{\partial \theta} \leq 0 \quad \text{and} \quad \frac{\partial A'_1(K_1, \theta)}{\partial \theta} \leq 0. \quad (14)$$

A possible illustration is when marginal abatement cost is an uncertain constant, i.e., when $A_1(K_1, \theta)$ is equal to $\alpha + g(\theta)K_1$ with $g' \leq 0$, a case examined by Baumstark and Gollier (2010). In that context, a small θ means at the same time a large marginal abatement cost and a large total abatement cost, and thus a low aggregate consumption. Thus, A'_1 and C_1 are anti-comonotone in that context, thereby demonstrating the following proposition.

Proposition 5. *Suppose that the speed of green technological progress θ is uncertain. If total and marginal abatement costs are comonotone (condition (14)), it is socially desirable that the growth rate of expected carbon price be smaller than the sum of the interest rate and the rate of decay of CO_2 .*

Up to this point, I only characterized the impact of uncertainty on the optimal growth rate of the carbon price. A more complete analysis would be to characterize its effect on the optimal abatement effort in the first period. This is a more difficult question. In order to address it, I simplify the problem by assuming that the marginal abatement cost in period 1 is constant but potentially uncertain: $A_1(K_1, \theta) = \theta K_1$. In that case, aggregate consumption in period 1 equals

$$C_1 = Y_1 - \theta \left(e^{-\delta} (Q_0 Y_0 - K_0) + Q_1 Y_1 - T \right).$$

Observe that in that case, the first period abatement K_0 has a role similar to saving in the standard consumption-saving problem. Each ton of CO_2 "saved" in the first period generates an increase in consumption by $R = \theta \exp(-\delta)$ in the second period, where R can be interpreted as the rate of return on savings. Suppose first that θ is certain. It is well-known in that case that the uncertainty affecting future incomes raises optimal (precautionary) saving if and only if the individual is prudent (Drèze and Modigliani (1972), Leland (1968), Kimball (1990)).¹¹ Applying this result to our context directly yields the following proposition. Notice that because the marginal abatement cost is certain, it must grow at the interest rate in this case.

Proposition 6. *Suppose that $A_1(K_1, \theta) = \theta K_1$ and that the marginal abatement cost θ is a known constant. Increasing risk on future production Y_1 or on the intertemporal carbon budget T increases the initial abatement effort K_0 if and only if the representative agent is prudent.*

¹¹ An individual is prudent if and only if the third derivative of u is positive.

When the marginal abatement cost is uncertain, the future return of abating more today becomes uncertain in that case. By risk aversion, this reinforces the willingness to abate in the first period because it also reduces the risk borne in the second period. Because of this second effect, prudence is sufficient but not necessary in this case.

Proposition 7. *Suppose that $A_1(K_1, \theta) = \theta K_1$ and that the marginal abatement cost θ is the only source of uncertainty. Increasing the risk affecting the marginal abatement cost θ raises the initial abatement effort K_0 if the representative agent is prudent.*

Proof: Consider two random variables, θ_1 and θ_2 , where θ_2 is riskier than θ_1 in the sense of Rothschild and Stiglitz (1970). Let $G_i(K_0) = H_i(K_0, K_1(K_0, Y_1, T))$ denote the corresponding objective function, as described by (10). Let K_{0i} denote the optimal initial abatement under distribution θ_i of the marginal abatement cost. The optimal abatement effort K_{01} under the initial uncertainty θ_1 satisfies the first-order condition

$$A'_0(K_{01})u'(Y_0 - A_0(K_{01})) = \beta E[\theta_1 u'(Y_1 - \theta_1 K_{11})], \quad (15)$$

where K_{11} is the optimal abatement effort in period 1 under the initial risk θ_1 , i.e., $K_{11} = K_1(K_{01}, Y_1, T)$. Because G_2 is concave in K_0 , I obtain that K_{02} is larger than K_{01} if and only if $G'_2(K_{01})$ is positive. Using condition (15), this condition can be written as follows:

$$E[\theta_2 u'(Y_1 - \theta_2 K_{11})] \geq E[\theta_1 u'(Y_1 - \theta_1 K_{11})]. \quad (16)$$

This is true for any Rothschild-Stiglitz risk increase if and only if function v is convex, where $v(\theta)$ equals $\theta u'(Y_1 - \theta K_{11})$ for all θ in the joint support of θ_1 and θ_2 . It is easy to check that

$$v''(\theta) = -2K_{11}u''(Y_1 - \theta K_{11}) + \theta K_{11}^2 u'''(Y_1 - \theta K_{11}). \quad (17)$$

Because K_{11} is positive and u'' is negative, we see that v is convex when u''' is positive. ■

4 Calibration

In this section, I calibrate the two-period model described in the previous section. A standard approach to climate policy in the western world is based on the hypothesis that the energy transition should be performed within the next 3 decades in order to remain below the 1.5°C objective with probability 1/2. I follow for example Metcalf (2018) to decompose the next 3 decades into two periods of 15 years, 2021-2035 and 2036-2050. I examine the case of the European Union (EU-28). I hereafter describe the calibration of this model. I assume a rate of pure preference for the present equaling $\rho = 0.5\%$ per year, and a constant relative risk aversion $\gamma = 3$.

4.1 Economic growth

The current annual GDP of EU-28 is around 19,000 billions US\$ (GUS\$). Assuming an annual growth rate of 1.4% per year over the period 2021-2035 yields a total production for this first period estimated at $Y_0 = 315,000$ billions US\$. The production Y_1 of the second period is uncertain. A key element of this paper is that the recommended returns of green

investments are compatible with the equilibrium returns of other assets in the economy, and with intertemporal social welfare. However, as is well-known, the CCAPM model that I use in this paper has been unable to predict observed asset prices when beliefs are normally distributed as assumed in Section 2. This model yields an interest rate that is too large and an aggregate risk premium that is too low.¹² In most of this paper, I use the resolution of these asset pricing puzzles that has been proposed by Barro (2006), who recognized the plausibility of infrequent large recessions that are not well represented in U.S. growth data. I follow the calibration proposed by Martin (2013). The change in log production during the second subperiod is equal to the sum of 15 independent draws of an annual growth rate x_i whose distribution compounds two normally distributed random variables:

$$\log\left(\frac{Y_1}{Y_0}\right) = \sum_{i=1}^{15} x_i \quad (18)$$

$$x_i \sim (h_{bau}, 1-p; h_{cat}, p) \quad (19)$$

$$h_{bau} \sim N(\mu_{bau}, \sigma_{bau}^2) \quad (20)$$

$$h_{cat} \sim N(\mu_{cat}, \sigma_{cat}^2). \quad (21)$$

With probability $1-p$, the annual growth rate is drawn from a "business-as-usual" normal distribution with mean $\mu_{bau} = 2\%$ and volatility $\sigma_{bau}^2 = 2\%$. But with a small probability $p = 1.7\%$, the annual growth rate is drawn from a "catastrophic" normal distribution with a large negative $\mu_{cat} = -35\%$ and a large volatility $\sigma_{cat}^2 = 25\%$. In Table 1, I describe the value of the parameters of the model that are used as a benchmark. The order of magnitude of the parameters of the production growth process is in the range of what has been considered by Barro (2006) and Martin (2013). It yields an annual trend of growth of 1.37% and an annual volatility of 6.12%.¹³ It also generates an expected production of $Y_1 = 387,000$ billions USD (GUS\$) in the second period.

4.2 Emissions, decarbonization and decay

The EU-28 currently emits 4.4 GtCO₂e per year. Under the Business-As-Usual (BAU), I assume that this flow is maintained over each of the 15 years of the first period, implying 66 GtCO₂e emitted in this scenario. When compared to the production Y_0 estimated above, this yields a carbon intensity of $Q_0 = 2.10 \times 10^{-4}$ GtCO₂e/GUS\$. Even without any mitigation policy, the world economy has benefitted from a natural reduction of the energy intensity of its global production over the recent decades. According to Clarke et al. (2014), the average rate of decline of the energy intensity has been approximately 0.8% per year between 1970 and 2010. This is why I assume in this calibration exercise that the carbon intensity in the

¹²See for example Kocherlakota (1996) and Cochrane (2017).

¹³It is interesting to compare the long run risk generated in this model to the one examined by Nordhaus (2018) and Christensen et al. (2018). They use a survey of a panel of experts to characterize the uncertainty in estimates of global output for the period 2010-2050. Experts were requested to estimate the average annual growth rate of the period. The resulting estimates were best fit using a normal distribution, with a mean of 2.59% and a standard deviation of 1.13%. This yields a standard deviation of $\log(Y_{2050}/Y_{2010})$ equaling $40 \times 1.13\% = 45.2\%$. This should be compared to the standard deviation of $\sqrt{40} \times 6.12\% = 38.7\%$ for this variable in my model. Thus, I assume long run output uncertainty whose intensity is similar to the sample in Nordhaus (2018).

second period goes down to $Q_1 = 1.85 \times 10^{-4}$ GtCO₂e/GUS\$ in the BAU. This implies an expected total emission of around 72 GtCO₂e in the second period under the BAU.

There exists an intense debate about the half-life of carbon dioxide in the atmosphere, and thus on its rate of natural decay. It appears that the carbon cycle is highly non-linear, and involves complex interactions between the atmosphere and different layers of the oceans. The existing literature on the half-life of carbon dioxide offers a wide range of estimates, from a few years to several centuries.¹⁴ I conservatively assume a rate of natural decay of CO₂ in the atmosphere of 0.5% per year. This implies a total expected emission net of the natural decay for the European Union over the period 2021-2050 in the BAU around 133 GtCO₂e.

4.3 Carbon budget

In the most recent report of the IPCC (IPCC (2018)), the goal of not exceeding a 1.5°C increase in temperature compared to the pre-industrial age is estimated to be compatible with a median carbon budget of 770 GtCO₂ in early 2018. Given that we have emitted around 40 Gt of greenhouse gases per year since then, I assume that this global carbon budget has now been reduced to 730 Gt. There is a debate about how to share this total carbon budget among the different countries. Let me take the conservative (and ethically sounded) approach of sharing the global budget on a per capita basis. Because the European Union is home for roughly 7% of the world population, I assume that EU-28 should be allocated a carbon budget of approximately 50 GtCO₂e. Let me further assume that four-fifth of this budget could be consumed between 2021 and 2050. This gives an expected carbon budget for EU-28 for that period equalling $\mu_T=40$ GtCO₂e. Compared to the global emission of 133 GtCO₂e, this represents a global abatement effort of 93 GtCO₂e, or a reduction of more than 70% of the global BAU emissions in the EU-28 during the next 3 decades.

There is of course much uncertainty about what will be the actual carbon budget that will emerge from the international negotiations in the next 3 decades, and from the resolution of the uncertainty about the intensity of climate change. I model this uncertainty by assuming that T is normally distributed with mean μ_T and standard deviation $\sigma_T = 10$ GtCO₂e.

4.4 Abatement costs

I assume that the abatement cost function is quadratic:

$$A_t(K_t) = a_t K_t + \frac{1}{2} b K_t^2. \quad (22)$$

An important element of the model is related to how the MAC changes with the ambition of the mitigation policy. The answer to this question is given by the MAC slope coefficient b . The researchers behind the MIT Emissions Prediction and Policy Analysis (EPPA, Morris et al. (2012)) have developed computable general equilibrium models with a very detailed energy sector. They have estimated the shadow price of carbon associated to various carbon budgets for different regions of the world, thereby generating regions-specific MAC curves. I used their analysis of the MAC curve for the European Union in 2020 to estimate that the MAC increases by 25 GUS\$/GtCO₂ whenever the annual abatement effort is increased by

¹⁴For a survey on this matter, see Archer et al. (2009).

parameter	value	description
ρ	0.5%	annual rate of pure preference for the present
γ	3	concavity of utility function
p	1.7%	annual probability of a macroeconomic catastrophe
μ_{bau}	2%	mean growth rate of production in a business-as-usual year
σ_{bau}	2%	volatility of the growth rate of production in a business-as-usual year
μ_{cat}	-35%	mean growth rate of production in a catastrophic year
σ_{cat}	25%	volatility of the growth rate of production in a catastrophic year
Y_0	315,000	production in the first period (in GUS\$)
δ	0.5%	annual rate of natural decay of CO ₂ in the atmosphere
Q_0	2.10×10^{-4}	carbon intensity of production in period 0 (in GtCO ₂ e/GUS\$)
Q_1	1.85×10^{-4}	carbon intensity of production in period 0 (in GtCO ₂ e/GUS\$)
μ_T	40	expected carbon budget (in GtCO ₂ e)
σ_T	10	standard deviation of the carbon budget (in GtCO ₂ e)
b	1.67	slope of the marginal abatement cost functions (in GUS\$/GtCO ₂ e ²)
a_0	23	marginal cost of abatement in the BAU, first period (in GUS\$/GtCO ₂ e)
μ_θ	2.31	expected future log marginal abatement cost in BAU
σ_θ	1.21	standard deviation of future log marginal abatement cost in BAU

Table 1: Benchmark calibration of the two-period model.

1 GtCO₂e. Expressed for a period of 15 years, this suggests $b = 1.67$ GUS\$/GtCO₂e². I assume that b is certain and constant over time.

Parameter a_t measures the MAC along the BAU scenario. For the first period, I estimate it by the price of carbon permits observed in the summer of 2018 on the EU-ETS market, around 23 GUS\$/GtCO₂e. The full elimination of the 66 GtCO₂e emitted in the first period would cost around 5,000 GUS\$, or 1.6% of GDP in the first period.

The MAC in the BAU during the second period is uncertain. Anticipating green innovations would suggest using a_1 smaller than a_0 , at least in expectation. By how much smaller remains an open question. In order to estimate the degree of uncertainty that surrounds abatement costs in the second period of our analysis, I have used a set of AIM models scrutinized by the Working Group III for the Fifth Report of the IPCC (Clarke et al. (2014)). In the associated database,¹⁵ I have collected the 374 estimations of carbon prices for 2030 that are in line with the objective of not exceeding 450ppm over the century. These estimates differ by the IAM model used for the estimation, and by the assumed technological progresses available at that time horizon. I provide the histogram of these MAC estimates for 2030 in Figure 2. The distribution of these estimates is heavily skewed to the right, which suggests using a lognormal distribution for $a_1 = \theta$. The standard deviation of the log MAC in this sample is equal to $\sigma_\theta = 1.21$.¹⁶ The standard deviation of the future MAC at the BAU is equal to 38 US\$/tCO₂e, which is in the range of the MAC uncertainty measured by Kuik

¹⁵<https://tntcat.iiasa.ac.at/AR5DB>

¹⁶Because these estimates are based on an ambitious abatement target, the mean value of the carbon price in this sample is not useful for the estimation of the expected MAC in the BAU.

et al. (2009) for a time horizon of 15 years.¹⁷

The trend of reduction in the MAC in my calibration is aligned with the assumption made by Nordhaus (2018) that the cost of the backstop technology declines at a rate of 0.5% per year. In the benchmark calibration, I assume that $\log(\theta)$ is normally distributed with mean $\mu_\theta = 2.31$ and standard deviation $\sigma_\theta = 1.21$. This yields an expected MAC in the BAU around 21GUS\$/GtCO₂e. The 8% reduction in the expected MAC under the BAU measures the green innovations that are expected to emerge in the next 15 years in the absence of new policies.

4.5 Results

I solved the first-order condition (11) numerically by using the Monte-Carlo method. I draw 350.000 random triplets (Y_1, θ, T) to estimate the expectation of the right-hand side of this equality, expressed as a function of K_0 . The optimal solution yields a specific stochastic consumption and mitigation path from which one can derive the equilibrium interest rate and the systematic risk premium:

$$r_f = \rho - \log \left(\frac{E[u'(C_1)]}{u'(C_0)} \right) \quad (23)$$

$$\pi = -\log \left(\frac{E[C_1 u'(C_1)]}{E[C_1] E[u'(C_1)]} \right). \quad (24)$$

One can also compute the stochastic path of the carbon prices (p_0, p_1) that supports this optimum. In Table 2, I describe the optimal solution of this problem under the calibration of the parameters described in Table 1. I obtain equilibrium asset prices that are in line with the real interest rate and the systematic risk premium that have been observed in the United States during the last century (Kocherlakota (1996)). Notice also that the expected optimal abatement is much larger in the second period than in the first one. This is partly due to the anticipation of a larger price of carbon in the second period. In expectation, the annualized growth rate of the carbon price equals 3.76%. This is much larger than the sum of the natural rate of decay of CO₂ and the interest rate, which is equal to 1.69%. This is due to the fact that at the optimum, the marginal abatement cost is positively correlated with aggregate consumption, as shown in Figure 3. In fact, the OLS estimation of the income-elasticity of the marginal abatement cost is $\phi \simeq 1.00$.¹⁸

As observed by Metcalf (2018), Aldy (2017) and Hafstead et al. (2017), carbon price predictability is the most important feature of a climate policy for the business community as it plans long-term investments in line with the energy transition. For example, Metcalf (2018) proposes to fix the annual growth rate of carbon price at 4% (plus inflation) as long as the path of emissions is in line with the objective. However, under uncertainty, the efficient growth rate of carbon price must be uncertain in this model because the resolution of the uncertainty affecting economic growth, green innovations and the carbon budget needs to be translated into a variable carbon price in the second period. In other words, it is not

¹⁷These authors performed a meta-analysis of MAC estimates in the literature, and observed a standard deviation of MAC of 27.9 and 52.9 euros per tCO₂e respectively for 2025 and 2050.

¹⁸Because consumption and marginal abatement costs are not log normal, equation (35) provides only a crude estimation of the optimal growth rate of the expected carbon price.

variable	value	description
K_0	31.10	optimal abatement in the first period (in GtCO ₂ e)
$E[K_1]$	66.30	optimal expected abatement in the second period (in GtCO ₂ e)
p_0	74.90	optimal carbon price in the first period (in US\$/tCO ₂ e)
$E[p_1]$	132.00	optimal expected carbon price in the second period (in US\$/tCO ₂ e)
g	3.76	annualized growth rate of expected carbon price (in %)
r_f	0.98	annualized interest rate (in %)
π	2.51	annualized systematic risk premium (in %)
ϕ	1.00	OLS estimation of the income-elasticity of the marginal abatement cost

Table 2: Description of the optimal solution in the benchmark case.

possible to be serious about the carbon budget constraint and, at the same time, to insure all economic agents against changes in the carbon price that is necessary to support the constraint. I represented the distribution of the carbon price p_1 and its annualized growth rate respectively in Figures 4 and 5. The standard deviation of this annualized growth rate is equal to 2.4% per annum. It reflects the uncertainties associated to the price of carbon necessary to satisfy the intertemporal carbon budget constraint. This constraint translates into an uncertain abatement effort in the second period, as described in Figure 6. Investment decisions in energy transition should take account of these uncertainties. The attractiveness of green investments should come from their expected return rather than from their reduced riskiness, something that cannot be guaranteed under a rigid carbon budget.

The high uncertainty affecting the second period carbon price is also the consequence of the cost-efficiency approach used in this paper. In the alternative cost-benefit approach, the absence of green innovation would be partially compensated by allowing emissions to grow.¹⁹ This is not possible if one takes the carbon budget constraint seriously. It implies that the carbon price has to grow faster under the cost-efficiency approach in this adverse scenario. Ex ante, this means that the carbon price uncertainty is larger.

What is the welfare cost of fighting climate change for the next three decades? To address this question, I measure welfare associated to a policy by the constant consumption level that generates the same discounted expected utility generated by that policy. Under the optimal carbon pricing rule, this "constant-equivalent consumption level" is equal to 330,020 GUS\$. This should be compared to the constant-equivalent consumption level of 332,560 GUS\$ that is obtained with the zero ambition strategy, i.e., when K_0 and K_1 are zero. This means that fighting climate change has an effect on intertemporal welfare that is equivalent to a permanent reduction of consumption by 0.763%.²⁰

4.6 The welfare cost of delaying action

We have seen in the introduction that most calibrations of cost-efficiency IAM models in the literature yield a growth rate of carbon price that is much larger than the interest rate.

¹⁹This observation implies that marginal abatement cost and marginal abatement benefits are positively correlated, as explained by Stavins (2019).

²⁰Of course, this measure does not take account of the benefits of reduced climate damages borne by future generations.

p_0	$E[p_1]$	g	K_0	$E[K_1]$	Welfare loss
23	179.85	13.71	0.00	95.16	1.044
30	173.36	11.69	4.19	91.27	0.970
40	164.08	9.41	10.18	85.71	0.886
50	154.80	7.53	16.17	80.16	0.826
60	145.53	5.91	22.16	74.60	0.785
70	136.25	4.44	28.14	69.05	0.765
75	132.00	3.76	31.10	66.30	0.763

Table 3: Cost of delaying the abatement effort. The initial price p_0 is arbitrarily selected between the BAU level (23 US\$/tCO₂) and its efficient level (75 US\$/tCO₂). The welfare loss measures the reduction (in %) in the constant welfare-equivalent consumption level compared to the no-abatement strategy. Units are as in Table 2.

Because these models assume no uncertainty, they imply a suboptimal allocation of the abatement effort over time, with a lack of effort in the short run, and too much effort in the long run. This may be due to the political command imposed to these calibrations. In this section, I am interested in measuring the welfare cost of this inefficiency. Our findings are summarized in Table 3.

If the EU maintains the price of permits at its 2018 level (23 US\$/tCO₂) for the next 15 years, it will be forced to increase it to almost 180 US\$/tCO₂ in expectation during the second period, which corresponds to an annual growth rate of 13.7%. This vastly inefficient intertemporal allocation of efforts yields a welfare loss that is equivalent to a permanent reduction of consumption by 1.04%. Compared to the efficient policy, this represents an increase in welfare loss by 28 basis points, from 0.76%. In short, this means that postponing the effort by 15 years has an effect on welfare which is equivalent to reducing consumption by a quarter of a percent during the next three decades, a 37% increase in the welfare cost of fighting climate change compared to the efficient policy.

It is noteworthy that the selection of an initial carbon price of 50 US\$/tCO₂, halfway between the BAU and the optimal carbon prices, yields a growth rate of expected carbon price of 7.5% per year, not far from what IAM models suggest. The welfare loss associated to this less inefficient policy is only 8% larger than when using the efficient policy.

4.7 Risk sensitivity analysis

Table 4 provides some information about the sensitivity of the optimum to the intensity of the exogenous risk of the model. The most interesting comparison to the benchmark is obtained when all sources of risk are switched off. Suppose that all standard deviations are reduced to zero, together with the probability of catastrophe. To preserve the mean growth rate of output, I reduced the mean growth rate to μ_{bau} to 1.37%. In this risk-free economy, we know that the efficient growth rate of carbon prices must be equal to the sum of the interest rate and the rate of natural decay. No risk premium should be included. However,

variable	benchmark	no risk	no cata	no macro risk	no tech risk	no budget risk
K_0	31.10	21.80	26.20	25.90	27.90	30.90
$E[K_1]$	66.30	73.10	69.30	69.40	69.30	66.50
p_0	74.90	59.40	66.80	66.30	69.60	74.60
$E[p_1]$	132.00	124.00	137.00	137.00	126.00	132.00
g	3.76	4.93	4.77	4.83	3.94	3.80
r_f	0.98	4.43	4.22	4.39	1.22	1.23
π	2.51	–	0.12	0.00	2.29	2.28
ϕ	1.00	–	0.66	-24.10	1.01	0.96

Table 4: Risk sensitivity analysis. The "no risk" context is obtained by equalizing all standard deviations to zero, by reducing the probability of catastrophe to zero, and by replacing μ_{bau} by 1.37% to preserve the expected growth rate of production as in the benchmark. The "no catastrophe" context is obtained by shifting the probability of catastrophe p to zero, and by reducing the trend of growth to μ_{bau} to 1.37%. The "no macro risk" context combines these changes with the shift of the volatility σ_{bau} to zero. In the "no tech risk" context, I switched σ_θ to zero compared to the benchmark. In the "no budget risk" case, I reduced σ_T to zero compared to the benchmark. Units are as in Table 2.

the absence of uncertainty switches off the precautionary motive to reduce the interest rate,²¹ which goes up to $r_f = 4.43\%$ in this context. This yields an efficient growth rate of carbon price of $r_f + \delta = 4.93\%$. The large discount rate implies that very little effort is made in the first period, with a low initial carbon price. In the benchmark calibration, the reason for why much of the mitigation effort is postponed comes from the fact that the MAC is positively correlated with aggregate consumption. In this alternative context with no risk, there is an even stronger argument for delaying the effort, namely, the absence of any precautionary motive to invest.

In the fourth column entitled "no catastrophe", I have solved the model by using the benchmark calibration except for the probability of catastrophe p that has been switched to zero, combined with a reduction of μ_{bau} to 1.37% in order to leave $E[Y_1]$ unchanged. This has the effect to raise the interest rate and to reduce the systematic risk premium to unrealistic levels. This observation justifies our choice of introducing macroeconomic catastrophes à la Barro in our calibration. In the next column, I fully eliminate the risk on Y_1 . The consequence of eliminating the macro risk is to generate a negative climate beta, as suggested by our theoretical results. However, because the systematic risk premium is a small 0.002% in the absence of macro risk (other than the carbon price risk), the growth rate of carbon price is only marginally smaller than the sum of the interest rate and the rate of decay.

In the last two columns of Table 4, I document the results of simulations in which risks on technological progress θ and on the carbon budget T are respectively switched off. Because these effects are relatively small, these results suggest that the main argument for a departure

²¹This precautionary motive to reduce the discount rate is best illustrated in the so-called "extended Ramsey rule" (equation (33) of the Appendix). For more details, see for example Gollier (2016).

variable	benchmark	doubling carbon budget	more green innovation	doubling cost curvature	Nordhaus	Stern
K_0	31.10	14.10	30.98	15.60	15.00	22.10
$E[K_1]$	66.30	42.20	66.37	40.90	41.40	37.50
p_0	74.90	46.60	74.75	75.00	48.00	59.80
$E[p_1]$	132.00	91.50	130.62	158.00	90.20	83.60
g	3.76	4.50	3.72	4.95	4.21	2.23
r_f	0.98	1.34	1.24	1.24	2.92	1.63
π	2.51	2.23	2.24	2.29	0.64	0.38
ϕ	1.00	1.80	1.00	2.23	1.82	1.98

Table 5: Parameter sensitivity analysis. In the "doubling carbon budget" scenario, I increase the expected carbon budget from $\mu_T = 40$ to 80 GtCO₂e. In the "more green innovation" scenario, I double the annual rate of reduction of the MAC in the BAU, so that μ_θ is reduced from 2.31 to 2.25. I double the curvature coefficient of the abatement cost function to $b = 3.34$ in the scenario entitled "doubling cost curvature". In "Nordhaus", I increase the rate of pure preference for the present from $\rho = 0.5\%$ to 1.5%, and I reduce relative risk aversion γ from 3 to 1.45. Finally, in "Stern", I reduce ρ to 0.1% and γ to 1. Units are as in Table 2.

of the Hotelling's rule $g = \delta + r_f$ comes from the macroeconomic uncertainty, not from technological risks or from carbon budget risks. These last two columns also tell us that the risk associated to climate change tends to reduce the equilibrium interest rate and the socially desirable risk-free discount rate. Indeed, recognizing that the emergence of mature green technologies and the level of the carbon budget are uncertain implies a lower interest rate, because of the enhanced precautionary motive to invest. It also implies a larger systematic risk premium. It can therefore contribute to the resolution of the classical asset pricing puzzles.

4.8 Parameter sensitivity analysis

I now turn to the sensitivity analysis related to the non-risk parameters of the model. In Table 5, I first double the expected carbon budget from $\mu_T = 40$ to 80 GtCO₂e. This increases the income-elasticity of the MAC and the efficient growth rate of carbon price. This implies a reduction of the carbon price in the first period by almost 40%. I also examined the effect of increasing the trend of reduction of the MAC in the BAU from around 0.5% to 1% per annum, but that reduces the optimal growth rate of carbon price only by 0.04%. The optimal climate policy is very sensitive to the curvature coefficient b of the abatement function. In line with Proposition 3 and the intuition that supports it, doubling the curvature more than doubles of the income-elasticity of the MAC, which in turn implies an increase in the optimal growth rate of carbon prices.

The last two columns of Table 5 are related to the Nordhaus-Stern controversy on the discount rate. The benchmark calibration was made compatible with observed asset prices. Following Barro (2006), I introduced macro catastrophes and I assumed a constant relative

risk aversion equaling $\gamma = 3$. I also used a rate of pure preference for the present equaling $\rho = 0.5\%$. These two coefficients are subject to an intense debate in our profession. Nordhaus (2018) uses a larger $\rho = 1.5\%$, whereas Stern (2007) uses a smaller $\rho = 0.1\%$. Both use a smaller γ of 1.45 for Nordhaus, and 1 for Stern. As illustrated by Table 5, this lower curvature of the utility function implies an equilibrium interest rate which is too large, and a risk premium which is too small. As expected, the Nordhaus' calibration of our collective preferences yields a much smaller initial carbon price and a much larger growth rate of carbon prices compared to the Stern's calibration.²²

4.9 An alternative approach: Epstein-Zin preferences

I used the Barro's extreme events argument in this calibration in order to solve the asset pricing puzzles that are inherent to the CCAPM model. In this section, I explore the alternative standard resolution of these puzzles that has been provided by Bansal and Yaron (2004). This "Long Run Risk" (LRR) model has two key ingredients. First, the representative agent is assumed to be endowed with Epstein-Zin recursive preferences, with a large degree of risk aversion, and a low degree of aversion to consumption fluctuations. Second, the growth rate of production is sensitive to a slow-moving state variable.

To adapt our model to this alternative resolution of the asset pricing puzzles, I perform two changes in our model. First, the objective function H now takes the following form:

$$H(K_0, K_1) = u(Y_0 - A_0) + e^{-\rho} u(\psi), \quad (25)$$

where ψ is the certainty equivalent of future consumption:

$$v(\psi) = Ev(Y_1 - A_1). \quad (26)$$

As is standard, I assume that u and v belong to the family of power functions, with $u(c) = c^{1-\gamma}/(1-\gamma)$ and $v(c) = c^{1-\alpha}/(1-\alpha)$. Parameter γ can be interpreted as the aversion to consumption fluctuations, which is the inverse of the elasticity of intertemporal substitution. Parameter α is relative risk aversion. The second change concerns the dynamic process governing the monthly growth rate of consumption x_i . Technically, I substitute the Barro's extreme events dynamics (19)-(21) by the following LRR dynamics:

$$x_{i+1} = \mu + z_i + \sigma \varepsilon_{i+1} \quad (27)$$

$$z_{i+1} = kz_i + \sigma_e e_{i+1}. \quad (28)$$

The slow-moving state variable z has an AR(1) dynamics with a persistency parameter k close to unity. It is assumed that ε and e are statistically independent $N(0, 1)$ random variables. I calibrate this model using the parameter values used by Bansal and Yaron (2004), who used a monthly frequency. This means that $\log(Y_1/Y_0)$ equals $x_1 + \dots + x_{180}$. Because market interest rates have experienced lower values since the publication of this paper, I have reduced the monthly rate of impatience from the 0.2% calibrated by Bansal and Yaron (2004) to 0.15%. The parameter values described in Table 6 replace those presented in the upper part of Table 1.

²²Under a cost-benefit approach, Nordhaus would also assume a larger carbon budget than Stern. This is not taken into account in the discussion based on an exogenous carbon budget.

	value	description
ρ	0.15%	monthly rate of pure preference for the present
γ	10	relative aversion to risk
α	2/3	relative aversion to consumption fluctuations
μ	0.15%	unconditional expected monthly growth rate of production
σ	0.78%	st. dev. of shocks to the monthly growth rate of production
σ_e	0.034%	st. dev. of shocks to predictable component of the monthly growth rate of production
k	0.979	persistence coefficient of the expected growth rate process

Table 6: Epstein-Zin calibration of the two-period model. All other parameters are as described in the bottom part of Table 1.

variable	value	description
K_0	33.44	optimal abatement in the first period (in GtCO ₂ e)
$E[K_1]$	68.79	optimal expected abatement in the second period (in GtCO ₂ e)
p_0	78.85	optimal carbon price in the first period (in US\$/tCO ₂ e)
$E[p_1]$	135.86	optimal expected carbon price in the second period (in US\$/tCO ₂ e)
g	3.63	annualized growth rate of expected carbon price (in %)
r_f	0.87	annualized interest rate (in %)
π	2.63	annualized systematic risk premium (in %)
ϕ	1.00	OLS estimation of the income-elasticity of the marginal abatement cost

Table 7: Description of the optimal solution of the long run risk model.

Because future consumption is not log-normally distributed, there is no analytical solution to the problem of maximizing H under the intertemporal carbon constraint (9). In Table 7, I describe the numerical solution to this problem. The bottom line of this exercise is that the LRR model generates an optimal carbon pricing schedule which does not differ much from our benchmark model. The two models share the same key observation that the CCAPM beta of efficient carbon prices is close to unity. They also share similar levels for the interest rate, the market risk premium and the growth rate of expected carbon prices. It is reassuring to conclude that the main messages of our analysis are independent of the strategy used to solve the classical asset pricing puzzles.

Other researchers have examined assets and carbon pricing in an Epstein-Zin framework. In Bansal et al. (2016), final consumption is also affected by the slow-moving average temperature (and climate damages), a second LRR factor in their model. Although both papers share the objective to explore the role of climate change in asset pricing, the channels of the climate impacts are radically different. In Bansal et al. (2016), asset prices are affected by climate damages, whereas they are affected by carbon prices and mitigation efforts in my model. Daniel et al. (2019) also characterize an optimal carbon pricing schedule with Epstein-Zin preferences. Rather than using a cost-efficiency approach as in this paper, they characterize the first-best solution with a climate damage function, considering a much longer time horizon of 300 years decomposed into 7 periods. Climate damages and mitigation costs

are uncertain, but they assume no uncertainty about productivity growth. Because I have shown that the absence of productivity growth uncertainty implies a negative carbon beta, it should not be a surprise that Daniel et al. (2019) obtain an initial carbon price that is relatively large, with a strongly *negative* trend of growth.²³ In their model contrary to mine, early green tech adoption provides a hedge against the long-term aggregate risk. I conjecture that adding some realistic productivity growth uncertainty into their model will increase the growth rate of carbon prices.

5 Conclusion

The future social and private benefits of most investments in renewable energy are uncertain by nature. One of their crucial social benefits is the reduction in emissions of CO₂, whose anticipated future pricing should be a key driver to induce market players to invest. Under a fixed intertemporal carbon budget constraint, the carbon price should send the right signal about the evolution of both the scarcity of emission permits and the cost of abatement efforts. For the sake of efficiency, it needs to be sensitive to macroeconomic and technological shocks. I have shown in this paper that, along the optimal mitigation path, the marginal abatement cost is positively correlated with aggregate consumption. To be more precise, I have shown that the MAC has a CCAPM-beta close to 1. Abating early generates a social benefit – the future MAC saved – whose risk profile that is not different from a claim on aggregate consumption. It should be priced accordingly, with a discount rate equaling the interest rate plus the aggregate risk premium. This means that the expected MAC and carbon price should grow at that rate. This provides the right compensation for early green technology adopters for the risk they take.

The renewable industry has often lobbied to obtain guarantees about future carbon prices, with the claim that it is a necessary condition for a rapid energy transition. This request is not substantiated. Rather than offering guarantees about future prices – a policy which would limit the quality of future price signals, one should offer them a larger expected rate of return for their investments in renewable energies, as a compensation for the risk that these investments yields. Again, this takes the form of planning a larger growth rate of expected carbon prices. Very risk-averse green investors should look for financial products that could hedge the carbon price volatility at market price. Of course, these recommendations rely on a credible institution able to implement a carbon pricing mechanism that is dual to our collective climate ambition.

²³Confirming a negative climate beta, they obtain an initial carbon price that is *increasing* in the market risk premium.

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Appendix: Proof of Proposition 2

Let $r_t = t^{-1} \log(E[B_t]/I_0)$ denote the expected return of an asset whose current price is I_0 and future benefit at date t is B_t . Using equation (1), the optimality condition can be written as follows:

$$e^{-r_t t} = e^{-\rho t} \frac{E[B_t u'(C_t)]}{u'(C_0) E[B_t]}. \quad (29)$$

Using equation (29), the risk-adjusted discount rate r_{ct} to discount a claim on aggregate consumption must satisfy the following efficiency condition:

$$\exp(-r_{ct} t) = \exp(-\rho t) \frac{E[C_t u'(C_t)]}{u'(C_0) E[C_t]}. \quad (30)$$

The systematic risk premium π_t is the extra expected rate of return of a claim on aggregate consumption over the interest rate that must compensate agents who accept to bear the macroeconomic risk:

$$\pi_t = r_{ct} - r_{ft}. \quad (31)$$

Under the two assumptions of the proposition, equation (2) implies that

$$1 = e^{-(\rho+\delta)t} E \left[\frac{A'_t u'(C_t)}{A'_0 u'(C_0)} \right] = e^{-(\rho+\delta)t} E [\exp(a'_t - \gamma c_t)].$$

Notice that our assumptions implies that $a'_t - \gamma c_t$ is normally distributed with mean $\mu_x - \gamma \mu_c$ and variance $(1 - \gamma \phi)^2 \sigma_c^2 + \sigma_w^2$. By Stein's Lemma, the above condition can then be rewritten as follows:

$$1 = \exp \left(\left(-\rho - \delta + \mu_p - \gamma \mu_c + 0.5(\phi - \gamma)^2 \sigma_c^2 + 0.5 \sigma_w^2 \right) t \right),$$

or, equivalently,

$$\mu_p + 0.5 \phi^2 \sigma_c^2 + 0.5 \sigma_w^2 = \delta + \rho + \gamma \mu_c - 0.5 \gamma^2 \sigma_c^2 + \phi \gamma \sigma_c^2. \quad (32)$$

In this economy, the following standard CCAPM formula for the risk-free interest rate can be derived from equation (??):

$$r_{ft} = r_f = \gamma + \gamma \mu_c - 0.5 \gamma^2 \sigma_c^2. \quad (33)$$

The systematic risk premium π_t is given by equation (31). Using Stein's Lemma twice to estimate r_{ct} given by equation (30) yields the following result:

$$\pi_t = \pi = \gamma \sigma_c^2. \quad (34)$$

Notice also that, using Stein's Lemma again, we have that the expected marginal abatement cost satisfies the following condition:

$$E \frac{A'_t}{A'_0} = E \exp(a'_t) = \exp \left(\left(\mu_p + 0.5 \phi^2 \sigma_c^2 + 0.5 \sigma_w^2 \right) t \right).$$

This implies that the growth rate g of expected marginal abatement cost is a constant given by

$$g = \frac{dEA'_t/dt}{EA'_t} = \mu_p + 0.5\phi^2\sigma_c^2 + 0.5\sigma_w^2.$$

Because in a decentralized economy, the marginal abatement cost is equal to the price of carbon in all states of nature and at all dates, g can also be interpreted as the growth rate of expected carbon price. Combining these properties implies that one can rewrite condition (32) as follows:

$$g = \delta + r_f + \phi\pi. \tag{35}$$

This concludes the proof of Proposition 2. ■

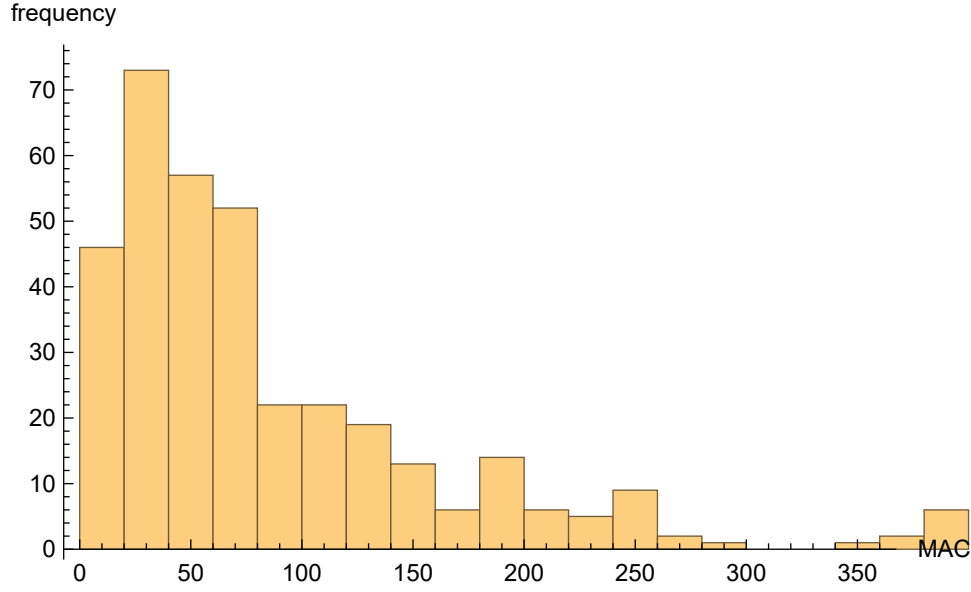


Figure 2: Histogram of the world marginal abatement costs for 2030 extracted from the IPCC database (<https://tntcat.iiasa.ac.at/AR5DB>). I have selected the 374 estimates of carbon prices (in US\$2005/tCO₂) in 2030 from the IAM models of the database compatible with a target concentration of 450ppm.

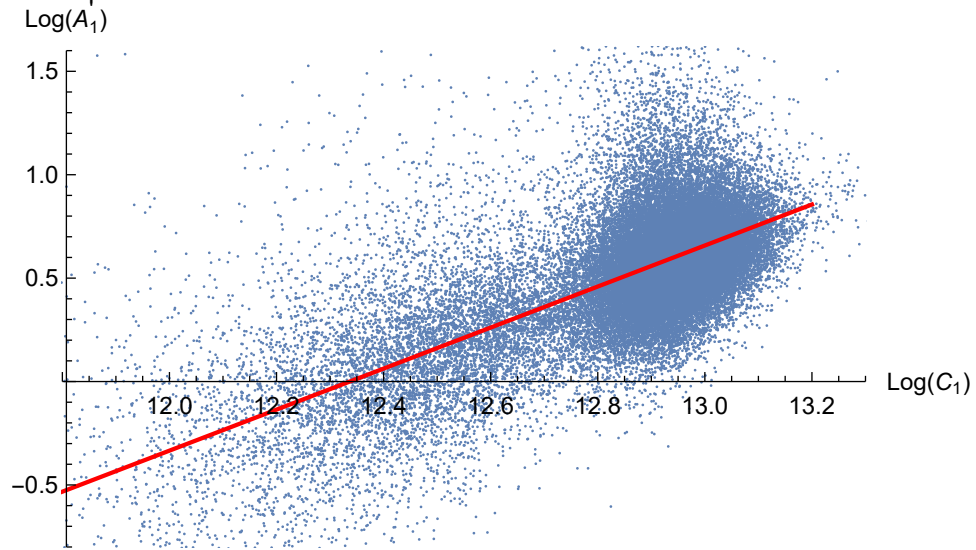


Figure 3: Monte-Carlo simulation under the benchmark case. For the sake of readability of the figure, I limited the simulation to 50.000 draws of the triplets (Y_1, θ, T) to estimate the optimal abatement strategy. The figure illustrates the positive statistical relation between log consumption growth and the log marginal abatement costs (and thus log carbon price) in the second period. The red curve depicts the OLS estimation in log-log.

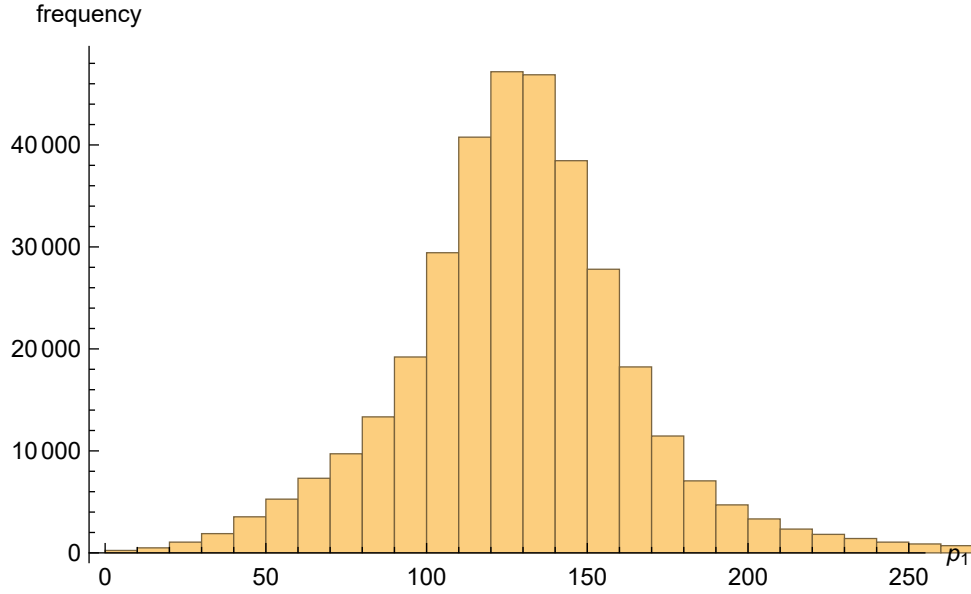


Figure 4: Empirical probability distribution of the carbon price p_1 (in US\$/tCO₂e) under the optimal abatement strategy in the benchmark calibration of the two-period model. The Monte-Carlo simulation uses a sample of 350.000 draws of the triplet (Y_1, θ, T) .

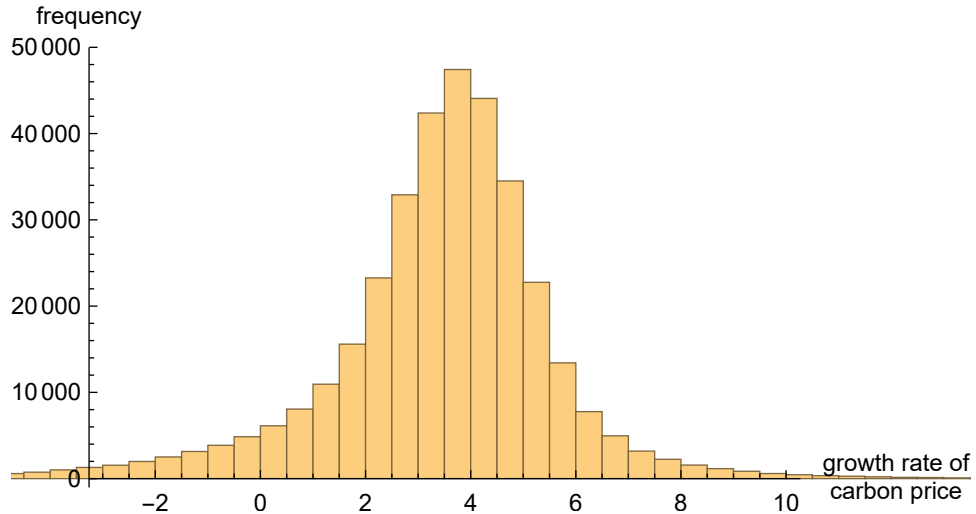


Figure 5: Empirical probability distribution of the annualized growth rate of carbon price under the optimal abatement strategy in the benchmark calibration of the two-period model. The growth rate is in percent per year. The mean growth rate is 3.36% and the standard deviation is equal to 2.5%.

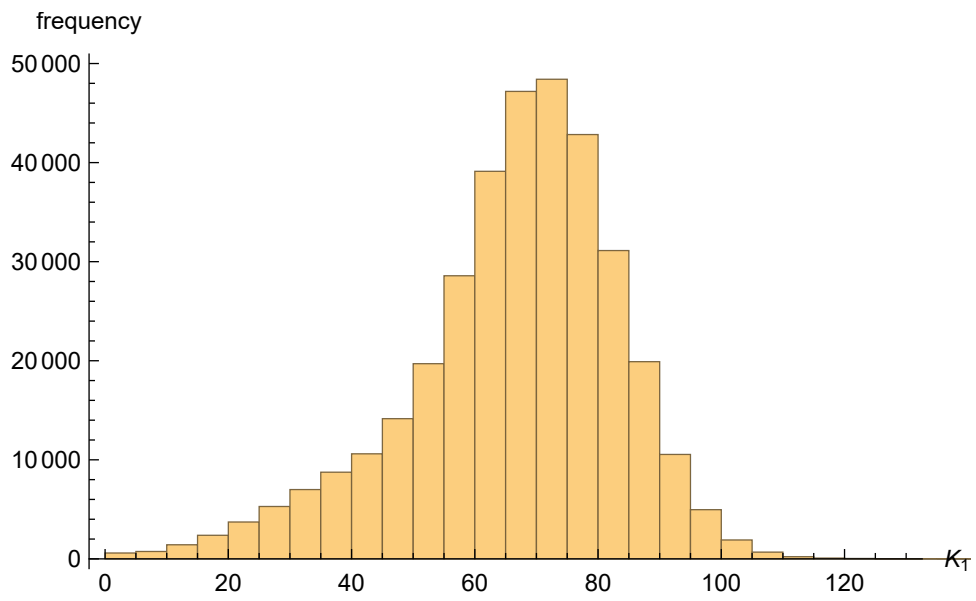


Figure 6: Empirical probability distribution of the abatement effort K_1 (in GtCO₂e) under the optimal abatement strategy in the benchmark calibration of the two-period model.